where

E_T, Thévenin equivalent e.m.f.;
R_T, Thévenin equivalent resistance of the system measured across the points in question in the absence of R and any source.

The ratio of perturbed to unperturbed voltage drops, and thus temperature differences by analogy, across the same points is then

$$\frac{\Delta T_p}{\Delta T_0} = \frac{\mathscr{R}_p}{\mathscr{R}_0}\,\frac{[\mathscr{R}_T + \mathscr{R}_0]}{[\mathscr{R}_T + \mathscr{R}_p]} = \frac{\left[\dfrac{\mathscr{R}_T}{\mathscr{R}_0} + 1\right]}{\left[\dfrac{\mathscr{R}_T}{\mathscr{R}_p} + 1\right]} \tag{B.2}$$

where the $\mathscr{R}$ values refer to thermal resistances

$$\mathscr{R} \equiv \frac{\ell}{AK} \tag{B.3}$$

with ℓ being the separation distance between the points and A the area of heat transfer.

Qualitatively, a perturbation can be characterized by the volume over which is applies and the amount by which the conductivity has been changed. Some insight into the behavior of the correction for limiting combinations of these characteristics can be evaluated using (B.3) if the problem is such that we may assume the following:

(a) Perturbation over a small volume $\Rightarrow \mathscr{R}_T \ggg \mathscr{R}_0$ or $\mathscr{R}_p$.
(b) Small perturbation in $K \Rightarrow \mathscr{R}_0 \approx \mathscr{R}_p$.
(c) Perturbation over a large volume $K \Rightarrow \mathscr{R}_T \lll \mathscr{R}_0$ or $\mathscr{R}_p$.
(d) Large change in $\Rightarrow \mathscr{R}_p \lll \mathscr{R}_0$ or the reverse.

Considering all combinations of the above,

Combination	$\Delta T_p/\Delta T_0$
Large volume, small change in K	$\approx 1{\cdot}0$
Large volume, large change in K	$\approx 1{\cdot}0$
Small volume, small change in K	$\approx 1{\cdot}0$
Small volume, large change in K	$\approx \mathscr{R}_p/\mathscr{R}_0 = K_0/K_p$

If the Thévenin equivalent resistances cannot be computed, the above considerations should provide some confidence as to the validity of the result.

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RECALCULATION OF THE SPALDING FUNCTION FOR CONTINUOUS AND SOURCE HEAT FLUX

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NOMENCLATURE

ρ, density;
τ, frictional stress at wall;
c_f, friction factor, $\tau/\frac{1}{2}\rho u_1^2$;
c_p, specific heat at constant pressure;
H, heat flux parameter, $\dot{q}''/\rho u_1 c_p \sqrt{(c_f/2)}$;
Pr, Prandtl number;
$\dot{q}''$, heat flux from wall;
S_q, Spalding function for continuous heat flux;
S'_q, Spalding function for source heat flux;
St, Stanton number;
T, temperature difference referred to free stream temperature;
u_1, free stream velocity;
u^+, non-dimensional velocity;
x^+, non-dimensional distance along the wall;
y^+, non-dimensional distance perpendicular to the wall.

1. INTRODUCTION

THE EXACT solution to the problem of predicting the temperature field within the fully developed turbulent boundary layer has been formulated by Spalding with the fundamental assumption of a particular "law of the wall" [1]. Numerical solutions for the cases of constant wall temperature and constant wall heat flux parameter have been obtained by Kestin and Gardner [2] and Smith and Shah [3].

Problems involving arbitrary distributions of temperature or heat flux can be treated by the method of superposition [3]. The merit of Spalding's approach over other methods lies in that its application is not restricted to flows of zero pressure gradient but is equally valid when a pressure gradient exists.

In the present paper a recalculation of the Smith and Shah solution for the case of constant heat flux parameter is presented. The present solution removes some errors in the earlier one, in that it tends to the Lighthill solution for laminar boundary layers at low values of x^+. An additional form of the Spalding function for a "source" heat flux (S_q') is derived from the solution and a new method of presentation is adopted which will give a more accurate and easier determination of the temperature at the wall and within the boundary layer.

2. METHOD OF SOLUTION

The solution was obtained as in [3] by numerical integration. The refinements over the earlier solution are:

(a) For low values of x^+ when the temperature boundary layer is still within the laminar sub-layer, the temperature at the wall was inserted from the Lighthill solution [4]:

$$S_q = 0{\cdot}615\,(x^+/Pr)^{-\frac{1}{3}}. \tag{1}$$

This method was adopted to ensure the continuity of the solution at low values of x^+.

(b) For higher values of x^+ the wall temperature was extrapolated from the field back to the wall in such a way as to allow for the curvature of the temperature distribution near the wall.

(c) The constants in the Spalding relation for the universal velocity profile

$$y^+ = u^+ + \frac{1}{E}\left\{e^{Ku^+} - 1 - Ku^+ - \frac{(Ku^+)^2}{2!} - \frac{(Ku^+)^3}{3!} - \frac{(Ku^+)^4}{4!}\right\} \tag{2}$$

were taken as $E = 12$ and $K = 0{\cdot}4$. These values are thought to give a better representation of the experimental data.

(d) The temperature gradient $\partial T/\partial x^+$ was mapped over the field. It will be shown that this gradient yields the "source" function S_q'.

3. PRESENTATION OF THE RESULTS

The results of the computation are given in Fig. 1(a–c) and Fig. 2(a–c). They are also available in tabular form.

(a) *Continuous heat flux function S_q*
The function S_q is defined as

$$S_q = St/\sqrt{(c_f/2)}. \tag{3}$$

The reciprocal $1/S_q$ is given for various values of u^+ in Fig. 1(a–c) for $Pr = 0{\cdot}7$, 1 and 7; the values at $u^+ = 0$ correspond to wall conditions. Values at $u^+ \neq 0$ correspond to various positions within the boundary layer.

S_q is obtained from the solution as

$$S_q = \frac{\dot{q}''}{\rho\, u_1 c_p \sqrt{(c_f/2)}} \times \frac{1}{T} = \frac{H}{T} \tag{4}$$

where $H \equiv \dot{q}''/\rho\, u_1 c_p \sqrt{(c_f/2)}$ is the constant heat flux parameter and T the temperature in the field. Thus the temperature at any point x^+, u^+ is given by

$$T = \frac{H}{S_q(x^+, u^+)}. \tag{5}$$

Figure 1 thus represents the temperature in the field due to a continuous heat flux of unit H starting at $x^+ = 0$.

Representative values of S_q from the present and the previous solution of [3] are compared in Table 1. Differences are sizeable at very low values of x^+ where the present solution tends to the Lighthill solution, and at high values of x^+, where the effect of the constants in the universal velocity profile becomes noticeable.

(b) *Heat flux source function S_q'*
The function S_q' is defined as

$$S_q' = \frac{\partial}{\partial x^+}\left(\frac{1}{S_q}\right). \tag{6}$$

This function is given for various values of u^+ in Fig. 2(a–c) for $Pr = 0{\cdot}7$, 1 and 7. S_q' is obtained from the temperature gradient field mapped in the solution. This can be seen by substituting $S_q = H/T$ in (6) giving

$$S_q' = \frac{\partial}{\partial x^+}\left(\frac{T}{H}\right) = \frac{1}{H}\frac{\partial T}{\partial x^+}. \tag{7}$$

The significance of the S_q' function will be apparent in the following section.

4. USE OF S_q AND S_q' FUNCTIONS

The function S_q will give the temperature at any point in the field (x^+, u^+) for a constant heat flux parameter H. However, since the expression for H includes the friction velocity, H will vary with x^+. Hence recourse will have to be made to the method of superposition, even for the simplest case, which is $\dot{q}'' =$ constant.

Consider the distribution of heat flux parameter H shown in Fig. 3. The temperature at plane b can be obtained by summing the temperature at b due to the strips 1–12, the strips 7–12 making a negative contribution. For each strip the heat flux parameter H is constant.

An alternative way would be to divide the heat flux field as in Fig. 4 into a series of heat flux sources of strength $H\,\Delta x^+$ and obtain the temperature at b by summing up the contributions from each source.

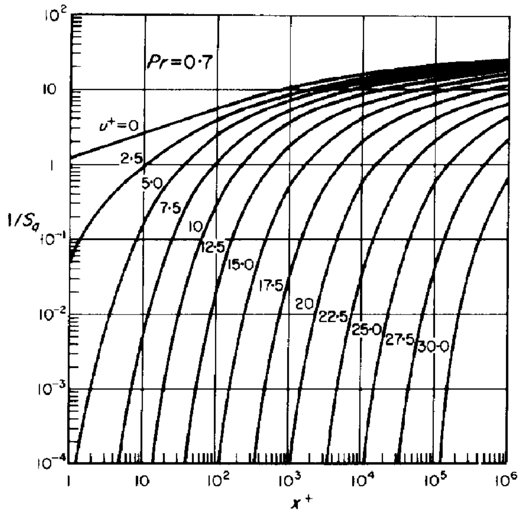

FIG. 1(a).

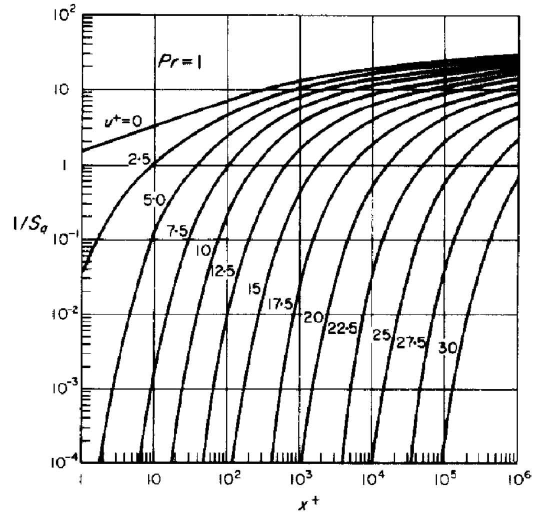

FIG. 1(b).

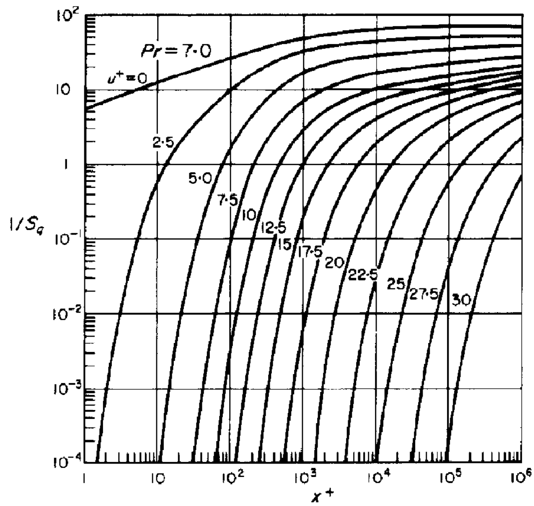

FIG. 1(c).

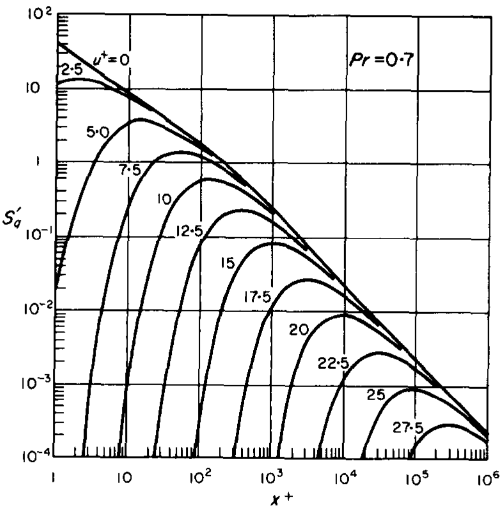

FIG. 2(a).

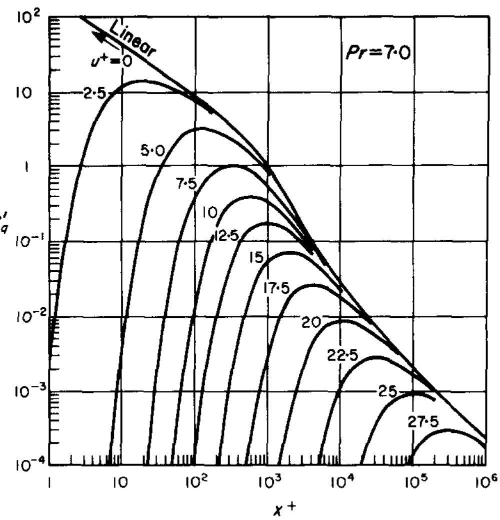

FIG. 2(b).

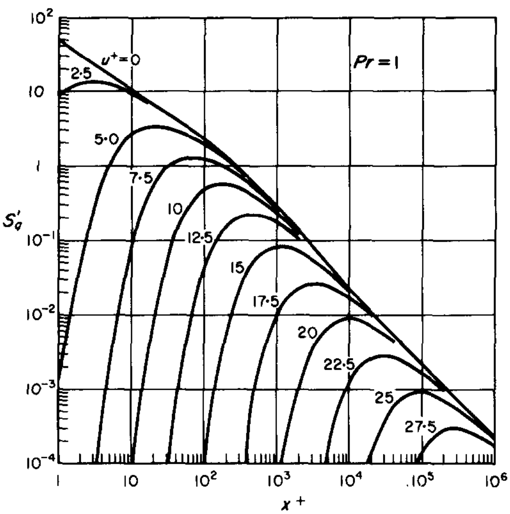

FIG. 2(c).

For the source shown in Fig. 5 the contribution will be

$$T = T_{ab} - T_{a'b}$$

$$= \frac{H}{S_{qab}} - \frac{H}{S_{qa'b}}$$

and if $ab - a'b$ (i.e. Δx^+) is small compared with ab

$$T = \frac{H\,\Delta x^+}{S_q^2}\,\frac{(S_{qa'b} - S_{qab})}{\Delta x^+}$$

$$= -\frac{H\,\Delta x^+}{S_q^2}\,\frac{\partial S_q}{\partial x^+}$$

$$= H\,\Delta x^+ \frac{\partial}{\partial x^+}\left(\frac{1}{S_q}\right).$$

Hence from (6)

$$T = H\,\Delta x^+\,S_q'.$$

The temperature at any point x^+, u^+ is given by

$$T = H\,\Delta x^+\,S_q'(x^+, u^+). \tag{8}$$

Figure 2 represents therefore the temperature in the field due to a source of unit strength at $x^+ = 0$.

Thus while $H/S_q(x^+, u^+)$ gives the temperature due to a

Table 1

x^+	$S_{q(u^+=0)}$								
	$Pr = 0.7$			$Pr = 1$			$Pr = 7$		
	First solution	Present solution	Difference %	First solution	Present solution	Difference %	First solution	Present solution	Difference %
1	0·801678	0·825806	+3·00	0·61424	0·650999	+5·98	0·172205	0·177902	+3·31
10	0·381063	0·384662	+0·94	0·29650	0·302894	+2·16	0·081847	0·082575	+0·89
10^2	0·179756	0·180335	+0·32	0·14177	0·142022	+0·18	0·038435	0·038563	+0·33
10^3	0·097158	0·095472	−1·73	0·07728	0·076000	−1·66	0·020749	0·020252	−2·40
10^4	0·065363	0·063730	−2·50	0·054413	0·052978	−2·64	0·017127	0·016352	−4·53
10^5	0·049523	0·048265	−2·54	0·042931	0·041702	−2·86	0·015713	0·015009	−4·49
10^6	0·039660	0·038713	−2·39	0·035321	0·034352	−2·74	—	0·013913	—

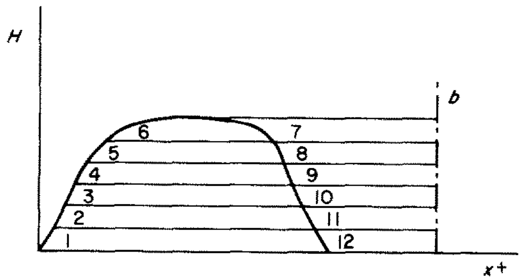

FIG. 3.

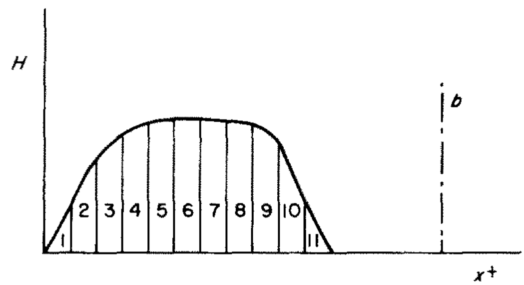

FIG. 4.

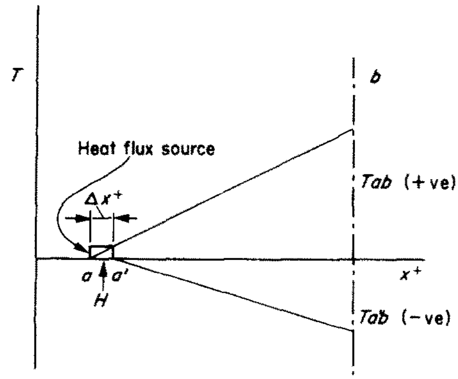

FIG. 5.

strip of constant H, x^+ upstream, $H\,\Delta x^+\,S'_q(x^+, u^+)$ gives the temperature due to a source of strength $H\,\Delta x^+, x^+$ upstream.

The advantage of the S'_q function is that it gives the temperature in the field at b as the arithmetic sum of a number of small increments, whilst the S_q function gives it the algebraic sum of a number of large increments.

The S_q functions must necessarily be used when the point at which the temperature distribution is required is within or near the heated section. For points further downstream the S'_q functions give a more accurate temperature distribution.

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